

COLLISION INDUCED GALAXY FORMATION

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ABSTRACT

We present a semi-analytical model in which galaxy collisions and strong tidal interactions, both in the field and during the collapse phase of groups and clusters help determine galaxy morphology. From a semi-analytical analysis based on simulation results of tidal collisions (Aguilar & White 1985), we propose simple rules for energy exchanges during collisions that allow to discriminate between different Hubble types: efficient collisions result in the disruption of disks and substantial star formation, leading to the formation of elliptical galaxies; inefficient collisions allow a large gas reservoir to survive and form disks. Assuming that galaxy formation proceeds in a $\Omega_0 = 1$ Cold Dark Matter universe, the model both reproduces a number of observations and makes predictions, among which are the redshifts of formation of the different Hubble types in the field. When the model is normalized to the present day abundance of X-ray clusters, the amount of energy exchange needed to produce elliptical galaxies in the field implies that they formed at $z \gtrsim 2.5$ while spiral galaxies formed at $z \lesssim 1.5$. The model also offers a natural explanation for biasing between the various morphological types. We find that the present day morphology-density relation in the field is well reproduced under the collision hypothesis. Finally, predictions of the evolution of the various galaxy populations with redshift are made, in the field as well as in clusters.

Subject headings: cosmology: theory - galaxies: formation - galaxies: morphologies - galaxies: kinematics and dynamics

1. Introduction

Gravitational interactions between galaxies are believed to play a major role in determining galaxy physical properties (Spitzer & Baade 1951; Toomre & Toomre 1972; Toomre 1974; Schweizer 1996; Schweizer & Seitzer 1992; Cole et al. 1994). For example, galactic interactions are likely to enhance star formation rates and could explain the intense star formation seen in IRAS galaxies. Indeed, these highly luminous infrared galaxies often show evidence of undergoing tidal interactions or mergers (Clements et al. 1996; Leech et al. 1994).

On theoretical grounds, models have shown that close encounters of galaxies stimulate cloud growth and star formation. Stellar and mass distributions in galaxies are likely to be altered by tidal collisions, and, as a result, the morphology of a galaxy may evolve. For example, the disks of spiral galaxies might be puffed up by such tidal encounters, and it has been suggested (Richstone 1976) that spiral galaxies might be converted into S0 or elliptical galaxies by such a mechanism. Following this idea, galaxy morphologies might be largely determined by the local properties of the environment in which they form and evolve, since tidal collisions between galaxies occur more frequently in denser regions of the universe.

Alternatively, galaxy morphologies may be related to intrinsic properties of the primordial fluctuation field. In the Cold Dark Matter (CDM) theory in which the density parameter $\Omega_0 = 1$, a bias must be invoked whereby the overdense regions contain galaxies, and the underdense regions (voids) are deficient in luminous galaxies, so that the luminous matter has stronger density correlations than the underlying dark matter. Early discussions of CDM used Gaussian fluctuations as described by the peaks formalism (Bardeen et al. 1986) and introduced a bias of the galaxy types with respect to mass by identifying 3σ primordial fluctuations with protoelliptical galaxies and 2σ fluctuations with protospirals in order to match the observed frequency and clustering of luminous galaxies (Blumenthal

et al. 1984; Evrard 1989; Evrard, Silk & Szalay 1990). Though this approach offered the advantage of reproducing several observed correlations, no physical mechanism was proposed to explain this intrinsic biasing until Dekel & Silk (1986) argued that 1σ fluctuations that satisfy the cooling criterion for galaxy formation (Rees & Ostriker 1977; Silk 1977) would be systematically of low-mass and shallow gravitational potential wells so that the rms density fluctuations should be especially vulnerable to disruption by supernovae explosions. A prediction of this model is that an extensive distribution of 'failed galaxies', identified as dwarf elliptical galaxies, would populate the low-density regions of the universe. However, it has been argued that observations strongly constrain the hypothesis that dwarf ellipticals can account for biasing (Binggelli 1989), the more luminous, observed dwarf ellipticals clustering together with the bright galaxies.

An alternative physical bias mechanism may naturally involve galaxy interactions as far as these interactions are assumed to trigger star formation. According to this approach, low-density regions of the universe should contain nascent or unborn galaxies, essentially gas clouds, since collisions are much less frequent in underdense regions. Conversely, in denser regions, substantial interactions at the epoch of formation would induce early star formation and subsequent collisions may redistribute the galactic stellar content and alter the galaxy morphology.

In this paper, we build a simple semi-analytical model of galaxy interactions to understand galaxy morphologies and how they relate to the properties of the environment and the fluctuation field at the epoch of formation. We shall show that simple collision rules provide an explanation of the morphological distribution of galaxies with respect to environment, as well as of fundamental correlations in their structural properties. Moreover, biasing of galaxies of different morphological types with respect to mass originates naturally in our scenario. The model is based on the results of N-body simulations of tidal collisions

(Aguilar & White 1985; AG85 hereafter). The paper is organized as follows: we first describe our model. Galaxy collisions are characterized in terms of the rate of change of binding energy induced in a given galaxy, at a given epoch, by tidal encounters with a set of background galaxies (§2). The cases for field and cluster galaxies are treated separately as the physics of collisions is quite different in these two environments. In §3, the total change of binding energy in a galaxy that went through a series of tidal collisions since its formation (we shall refer to this quantity as the ‘collision factor’) is computed and characterizes the collision history of the galaxy. Scalings of the collision factor with local galaxy density are obtained. In §4, we propose a phenomenological definition of galaxy morphological types that can be easily expressed as conditions on the collision factor (§4.2). We show (§4.3) that these conditions are in turn conditions on the formation redshift of galaxies, and we define redshift cuts, in the space of formation redshifts, that delineate spiral, S0 and elliptical galaxies. Sections 5 and 6 are devoted to normalizing our model to various sets of observations, assuming CDM initial conditions. Predictions of the model are given in the remaining sections. The model predicts the redshift of formation (§7) as well as the relative bias between galaxies of various morphological types (§8) in the field. The morphology-density relation is well reproduced by the model (§9). The predictive power of the model is illustrated in §10, where the evolution of the morphological populations with redshift is predicted, under CDM initial conditions, in the field as well as in denser environments. Throughout the paper, the case for an Einstein-de Sitter ($\Omega_0 = 1$) with a Hubble constant $H_0 = 50$ km/s/Mpc is assumed.

2. Characterization of galaxy collisions

We characterize galaxy collisions in terms of the rate of change of binding energy E of a galaxy colliding with an other. This can be expressed in a general form for a test galaxy

(of mass M and radius R) interacting with a set of background galaxies (of mass M_p and radius R_p) with relative velocity v as (Richstone 1975; AG85):

$$\dot{\Delta} = n_p v R^2 f_E(M_p/M, V/v) \quad (1)$$

where $\dot{\Delta}$ stands for $d \ln E / dt$. n_p is the number density of background galaxies, and V the internal velocity dispersion of the test galaxy. The dimensionless function f_E is plotted in figure 8 of AG85 for equal masses galaxies. It is easy to see from this figure that f_E scales roughly as the square of the ratio V/v so that equation (1) simply reads as:

$$\dot{\Delta} = f R^2 \left(\frac{V}{v}\right)^2 v n_p \quad (2)$$

where f is the slope of the function f_E and is equal to $f \approx 30$ (AG85; figure 8). For unequal galaxy masses, two modifications of (1) may be anticipated: 1) R should be replaced by $R_>$, where $R_> = \max(R, R_p)$, and 2) powers of the mass ratio M/M_p may appear. For a test galaxy (R, M) perturbed by a set of background galaxies (R_p, M_p) with distribution $dn_p = \eta(M_p, z) dM_p$, the perturbation δV on the stellar velocity field can be estimated from the impulse approximation (Spitzer 1958) and straight line trajectories (provided $v \gg V$) to be:

$$\frac{\delta V}{V} \sim \frac{G M_p}{v V} \frac{R}{p^2} \quad (3)$$

where p is the impact parameter which for the dominant contribution to the rate of energy exchange can be taken to approximately equal $R_>$. Using the virial theorem for the test galaxy, equation (3) becomes:

$$\frac{\delta V}{V} \sim \frac{V}{v} \frac{R^2}{R_>^2} \frac{M_p}{M} \quad (4)$$

For equal mass galaxies, we have from (4) that $\delta V/V \sim V/v$ so that $\dot{\Delta}$ scales as $(\delta V/V)^2$ - equation (2). We assume this still holds for unequal masses with $\delta V/V$ given by (4). The rate for energy exchange now reads as:

$$\dot{\Delta} = \int_{M_p} f R_>^2 \left(\frac{V}{v}\right)^2 \left(\frac{M_p}{M}\right)^2 \left(\frac{R}{R_>}\right)^4 v \eta(M_p, z) dM_p \quad (5)$$

where the integral is performed over the masses of background galaxies. As equation (5) relies on the impulse approximation and straight line trajectories, it is valid only for $v \gg V$. This latter condition is fulfilled in galaxy clusters, for example, as the relative velocity between galaxies is expected to be of the order of the cluster velocity dispersion $\sigma \sim 1000$ km/s, whereas the internal velocity dispersion of galaxies is of the order of $\sim 200 - 300$ km/s. In the field, the relative velocity of colliding galaxies may be of the same order as their internal velocity dispersion and the relative trajectories can not be approximated by straight lines. The focusing due to mutual attraction can be easily incorporated into equation (5). If p and v are the initial impact parameter and relative velocity, energy conservation during the collision implies that, in the frame of the reduced particle of mass $\mu = M_p M / (M_p + M)$:

$$\frac{1}{2}\mu v^2 = \frac{1}{2}\mu v_{col}^2 - \frac{GMM_p}{R_{col}} \quad (6)$$

where v_{col} is the relative velocity at closest approach R_{col} , and G is the gravitational constant. Conservation of momentum per unit mass L implies $L = pv = R_{col}v_{col}$ from which we get:

$$p^2 = R_{col}^2 \left[1 + \frac{2G(M + M_p)}{R_{col}v^2} \right] \approx R_{col}^2 \left[1 + \frac{2G(M_{>})}{R_{col}v^2} \right] \quad (7)$$

Using the virial theorem for the largest of the two colliding galaxies (of mass $M_{>}$ and internal velocity dispersion $V_{>}$), we have:

$$p^2 \approx R_{col}^2 \left[1 + \left(\frac{V_{>}}{v} \right)^2 \right] \quad (8)$$

Taking $R_{col} \sim R_{>}$, gravitational focusing roughly amounts to multiplying $R_{>}^2$ in equation (5) by the factor

$$1 + \left(\frac{V_{>}}{v} \right)^2 \quad (9)$$

For a similar reason, the relative velocity entering equation (3) is increased by the square root of (9). However, the energy taken out of the relative motion to be injected into the internal stellar motions lowers the relative motion in a similar way. If $v \sim V$, the two effects

nearly cancel each other. We do not attempt to model these fairly complex processes, but simply use the rate (5) modified by the factor (9). The scaling with mass of the latter is needed in order to take into account the strong attraction produced by large galaxies, and will be important for the scaling of the rate of change of energy with mass and redshift.

We now integrate (5) over the masses of the background galaxies. We use $R_{>} = R$, the radius of the test galaxy, so that the test galaxy is larger, on average, than the background galaxies. We define $\int M_p \eta(M_p) dM_p = \varepsilon \bar{\rho}$, where $\bar{\rho}$ is the mean density in the universe and ε is the mass fraction in galaxies, so that we get:

$$\dot{\Delta} \approx f R^2 \left(\frac{V}{v}\right)^2 \left(\frac{M}{\bar{M}}\right)^{-2} \frac{\varepsilon \bar{\rho} v}{\bar{M}}, \quad v \gg V \quad (10)$$

and

$$\dot{\Delta} \approx f R^2 \left(\frac{V}{v}\right)^4 \left(\frac{M}{\bar{M}}\right)^{-2} \frac{\varepsilon \bar{\rho} v}{\bar{M}}, \quad v \sim V \quad (11)$$

where \bar{M} in equation (10) and (11) is defined by:

$$\bar{M} \equiv \frac{\int M_p^2 \eta(M_p) dM_p}{\int M_p \eta(M_p) dM_p}. \quad (12)$$

\bar{M} is dominated by the larger masses due to the M_p factors: hence, we expect \bar{M} to be close to the mass of the larger galaxies.

To summarize this section, we have obtained scaling relations for the rate of change of binding energy as a function of the various parameters characterizing the colliding galaxies. Table 1 gives a list of the names and meaning of the parameters used so far. Table 2 summarizes the results we shall build our model on in the next sections, and the various assumptions used.

3. Collision history of galaxies

The rate of change of binding energy for a galaxy interacting with surrounding galaxies, between redshift z and $z + dz$ is

$$d \ln E = \dot{\Delta} \frac{dt}{dz} dz, \quad (13)$$

where $\dot{\Delta}$ has been given by (10) and (11) in the previous section. Integrating (13) over time from the redshift of formation z_{nl} of the galaxy up to redshift z yields the total increase of binding energy due to a series of collisions experienced by the galaxy during its lifetime. This integrated quantity characterizes what we shall call the ‘collision history’ of a galaxy. To evaluate this quantity, we need to use the relevant scaling with redshift of the various quantities entering equations (10), (11) and (12), which have been listed in Table 1. These scalings will be different in cluster and field environments. We shall therefore distinguish from now on the cases for cluster and field galaxies. We remind the reader that all the results given in this paper are for an Einstein-de Sitter universe, that is $\Omega_0 = 1$, and for $h = 0.5$, where h is the Hubble constant in units of 100 km/s/Mpc.

3.1. Redshift evolution of \bar{M}

By construction (equation (12)), \bar{M} depends on the mass distribution $\eta(M)$ of the background galaxies. $\eta(M)$ evolves with redshift and so does the average mass \bar{M} in our model. We evaluate the scaling of \bar{M} with redshift in this section.

3.1.1. Press & Schechter mass function

With the assumption that the initial fluctuations are Gaussian distributed, the mass function of dark halos can be obtained from the condition that a fluctuation is non-linear

at a given mass scale, but not at an immediately larger scale (Press & Schechter 1974; Schaeffer & Silk 1985). The number density of non-linear condensations of mass between M and $M + dM$ at redshift z is, in an Einstein-de Sitter universe:

$$\eta(M, z) = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M^2} \frac{d \ln \sigma^{-1}(M)}{d \ln M} \frac{\delta_c(1+z)}{\sigma(M)} \exp \left[-\frac{1}{2} \left(\frac{\delta_c(1+z)}{\sigma(M)} \right)^2 \right] \quad (14)$$

where ρ_0 is the present day average density of the universe and δ_c is the linearly extrapolated threshold on the density contrast required for structure formation. We adopt the canonical value of the spherical model (Lemaître 1933; Peebles 1980; Gunn & Gott 1972), i.e.

$\delta_c \approx 1.68$, and the average density of an object collapsing at redshift z is $178\rho_0(1+z)^3$. The relative mass fluctuation $\delta M/M$ in a volume that contains a mass M in the linear stage enters equation (14) through its variance

$$\sigma(M) = \left\langle \left(\frac{\delta M}{M} \right)^2 \right\rangle^{\frac{1}{2}}, \quad (15)$$

which is known as soon as the primordial fluctuation spectrum is specified. In the following, we shall use the Cold Dark Matter (CDM) spectrum that can be parameterized (e.g., Narayan & White 1988) as a function of the comoving scale R , corresponding to the mass scale M , as

$$\sigma_{CDM}(M) = 16.3(1 - 0.3909R^{0.1} + 0.4815R^{0.2})^{-10}/b \quad (16)$$

where the bias parameter b has been introduced and is specified by the amplitude of underlying matter fluctuations at $8h^{-1}$ Mpc, σ_8 . The parameterization (16) is given for $h = 0.5$, a value that we shall adopt throughout the paper.

At a given redshift z , we shall now require that gas cooling occurs after virialization to allow for gas fragmentation and star formation (Silk 1977, Rees & Ostriker 1977). This is important for evaluating the number density of background galaxies, as only a subset of all mass condensations counted for by the Press & Schechter prescription (14) are actual galaxies.

3.1.2. The cooling constraint

The condition for cooling to be effective is a condition on the relative importance of the cooling time-scale and the dynamical time-scale. Various processes contribute to gas cooling. Assuming that during the collapse, the gas is shock heated and reaches virial temperature before settling into a disk, we are led to consider only those cooling processes that are efficient at temperatures of the order of the virial temperature. For simplicity, we assume that line cooling is the dominant mechanism, and shall adopt the following cooling function for gas temperatures of the order $\sim 10^5 - 10^6$ K (e.g., Sutherland & Dopita 1993):

$$\Lambda(T) \approx 2.5 \times 10^{-21} T^{-1/2} n^2 \quad \text{erg cm}^{-3} \text{s}^{-1} \quad (17)$$

where n is the particle density of the gas within a galaxy and T the virial temperature. Zero metallicity has been assumed. The typical cooling time-scale is then

$$t_{cool} \sim 3 \frac{n k T}{\Lambda(T)} \approx 5.5 \times 10^6 \left(\frac{n}{1 \text{cm}^{-3}} \right)^{-1} \left(\frac{T}{10^6 \text{K}} \right)^{3/2} \quad \text{yr} \quad (18)$$

where k the Boltzmann's constant. If the gas makes up a constant fraction \mathcal{F}_B of the total mass M of the galaxy, and is uniformly distributed within the virial radius R , then the gas density n is

$$n = \frac{3}{4\pi\mu m_p} \frac{\mathcal{F}_B M}{R^3} = 1.6 \times 10^{-3} \left(\frac{M}{10^{12} M_\odot} \right) \left(\frac{R}{100 \text{kpc}} \right)^{-3} \left(\frac{\mathcal{F}_B}{0.1} \right) \quad \text{cm}^{-3} \quad (19)$$

where m_p is the proton mass and μ the mean molecular weight, that for an hydrogen-helium plasma with primordial abundances is $\mu \approx 0.6$. The temperature of the gas is obtained from the virial equation:

$$kT \approx \mu m_p \frac{V^2}{3} \approx \mu m_p \frac{GM}{5R} \quad (20)$$

where V is the 3-D galaxy velocity dispersion. In (20), we have assumed that the galaxy is spherical and the gas is homogeneously distributed within R . Plugging (19) and (20) into

(18), we obtain the following cooling time-scale:

$$t_{cool} \approx 1.7 \times 10^9 \left(\frac{\mathcal{F}_B}{0.1} \right)^{-1} \left(\frac{M}{10^{12} M_\odot} \right)^{1/2} \left(\frac{R}{100 \text{kpc}} \right)^{3/2} \text{ yr} \quad (21)$$

The dynamical time-scale can be estimated from the time taken for the galaxy to collapse after turn around:

$$t_{dyn} \approx \frac{\pi}{2} \sqrt{\frac{R_{ta}^3}{2GM}} \approx 1.5 \times 10^9 \left(\frac{M}{10^{12} M_\odot} \right)^{-1/2} \left(\frac{R}{100 \text{kpc}} \right)^{3/2} \text{ yr} \quad (22)$$

where $R_{ta} \approx 2R$ is the turn around radius, and G the gravitational constant. For efficient star formation, we require (21) to be smaller than (22) implying:

$$M < M_* \approx 9 \times 10^{11} M_\odot \quad (23)$$

where M_* is a critical mass whose value is fixed by the physics of cooling.

For the purpose of evaluating the number density of background galaxies, we shall use the constraint (23) that gives an upper limit of the mass of galaxies. In figure 1, we plot the average mass \bar{M} , computed from (14), as a function of redshift (solid line) when the cooling constraint is applied. Integrals entering (12) have been computed using a minimum mass $M_{inf} = 10^{10} M_\odot$. For comparison, \bar{M} obtained from (14) with no cooling constraint is also plotted (dashed line). When the cooling constraint is used, the average mass \bar{M} exhibits only a weak dependence on redshift over the range $z \lesssim 3 - 4$, much weaker than when no cooling is used. Moreover, the average mass is found to be of the order of M_* . We shall adopt this value hereafter and take \bar{M} constant with redshift for $z \lesssim 3 - 4$.

3.2. Field galaxies

The average density of the universe in equation (11) scales with redshift as

$$\bar{\rho} = \rho_0 (1 + z)^3 \quad (24)$$

where ρ_0 is the average density of the universe at the present epoch. We assume that the average density in galaxies reflects the density of the universe at the epoch of formation z_{nl} within a universal factor (that depends on the properties of the collapse). We then can relate the radius R of the test galaxy to its mass M as

$$R = R_* \left(\frac{M}{M_*} \right)^{1/3} \left(\frac{1 + z_{nl}}{1 + z_*} \right)^{-1} \quad (25)$$

For simplicity, we have expressed the relevant quantities for the test galaxy in terms of the same quantities for the typical M_* galaxy introduced in the previous section. A typical M_* galaxy has radius R_* , mass M_* , internal velocity dispersion V_* , and form at redshift z_* .

Using equation (25) and the virial theorem, we obtain the following scaling of the internal velocity dispersion of the test galaxy V with redshift:

$$V = V_* \left(\frac{M}{M_*} \right)^{1/3} \left(\frac{1 + z_{nl}}{1 + z_*} \right)^{1/2} \quad (26)$$

We finally need to know how the relative velocity v evolves with redshift. We assume that the mean relative velocity of colliding galaxies can be inferred from the peculiar motions of galaxies in the linear regime at redshift z :

$$v = a \frac{dx}{dt} = v_0 (1 + z)^{-1/2} \quad (27)$$

where x is a comoving coordinate, a is the scale factor of the universe, and v_0 the present-day galaxy peculiar velocity in the field. This assumption is valid at high z in the linear regime. An accurate scaling at low redshift should take into account pairwise velocity on small scales in the non-linear regime; however, (27) introduces only a weak dependence on redshift, and we shall adopt this scaling in the following. Plugging equations (24),(25),(26) and (27) into equation (11), we get

$$\dot{\Delta} = \varepsilon f R_*^2 \frac{v_0 \rho_0}{M_*} \left(\frac{V_*}{v_0} \right)^4 (1 + z)^{9/2}. \quad (28)$$

The redshift integration of equation (28) from the redshift of formation z_{nl} of the test galaxy then yields the total increase of binding energy due to collisions, up to an epoch

characterized by redshift z . In the following, we shall call this quantity the ‘collision factor’ and denote it Δ . In an Einstein-de Sitter universe, redshift and time are related through:

$$dt = -H_0^{-1}(1+z)^{-5/2}dz \quad (29)$$

where H_0 is the present day value of the Hubble constant. Integration of (28) over redshift thus leads to:

$$\Delta^{field}(z) \equiv \int_{z_{nl}}^z \dot{\Delta} \frac{dt}{dz} dz = \Delta_*(1+z_{nl})^3 \left[1 - \left(\frac{1+z}{1+z_{nl}} \right)^3 \right] \quad (30)$$

where Δ_* is the dimensionless constant

$$\Delta_* = \varepsilon(f/3) R_*^2 \left(\frac{V_*}{v_0} \right)^4 \frac{v_0}{H_0} \frac{\rho_0}{M_*}. \quad (31)$$

A numerical estimate of (31) can be obtained by fixing the parameters describing a typical M_* galaxy today, and the relative velocity v_0 .

3.3. Cluster Galaxies

For field galaxies, collisions are only important at early times when the density is large. Similarly, we expect little effect in the lower density regions of clusters. In the very high density regions of rich clusters, however, the energy exchange due to collisions is high and is given by equation (10) with $\bar{\rho}$ replaced by the local density ρ and v by the local galaxy velocity dispersion of the cluster. Also, in the focusing factor (9), the term $(V_>/v)^2$ that was dominant in the field can now be neglected. Using equations (25) and (26), equation (10) thus becomes:

$$\dot{\Delta} = f \frac{\epsilon \rho v}{M_*} R_*^2 \left(\frac{V_*}{v} \right)^2 \left(\frac{M}{M_*} \right)^{-2/3} \left(\frac{1+z_{nl}}{1+z_*} \right)^{-1}. \quad (32)$$

Performing the same integration over redshift as in the previous subsection, now for cluster environment, yields the change of binding energy for a cluster galaxy from the epoch z_{nl} up

to redshift z :

$$\Delta^{cluster}(z) = \Delta_c (1 + z_*)^{-3/2} \left(\frac{M}{M_*}\right)^{-2/3} \left(\frac{V_*}{v}\right) \left(\frac{\rho}{\rho_0}\right) \left(\frac{1 + z_{nl}}{1 + z_*}\right)^{-5/2} \left[\left(\frac{1 + z}{1 + z_{nl}}\right)^{-3/2} - 1\right] \quad (33)$$

with

$$\Delta_c = \frac{2}{3} \frac{f}{H_0} R_*^2 \frac{\varepsilon \rho_0}{M_*} V_*, \quad (34)$$

where the average density of the universe, ρ_0 , has been introduced for convenience.

3.4. Scaling of $\Delta^{cluster}$ with density

In equation (33), both ρ and v can be related to the local galaxy number density n , provided a model is assumed for the density distribution of the cluster. Let us consider for instance the following cluster profile:

$$\rho(R) = \rho_c \left(\frac{R}{R_c}\right)^{-p} \quad (35)$$

where ρ_c is the cluster density at a radius R_c from the center, and depends on the mass and redshift of formation of the cluster. By use of the virial theorem and (35), the velocity dispersion v can be expressed as a function of the cluster density ρ :

$$v = v_c \left(\frac{\rho_c}{\rho_0}\right)^{\frac{1}{p}} \left(\frac{\rho}{\rho_0}\right)^{\frac{p-2}{2p}} \quad (36)$$

where v_c is the cluster velocity dispersion at radius R_c . As expected for $p = 2$, the velocity dispersion of a given cluster is constant and independent of the density ρ . This velocity, however, is not the same for all clusters since it depends on ρ_c :

$$v = v_c \left(\frac{\rho_c}{\rho_0}\right)^{1/2} \quad p = 2. \quad (37)$$

The collision factor (33) scales, in this simple case, as

$$\Delta^{cluster} \propto \rho \rho_c^{-1/2}. \quad (38)$$

We can relate the cluster density ρ to the local galaxy number density n by assuming that galaxies trace mass, that is $n \propto \rho$. We shall however adopt a somewhat more realistic model for the cluster density profile, namely an isothermal β model (Cavaliere & Fusco-Femiano 1976) in which the galaxy number density in the cluster is related to the gas density profile $\rho_g(R)$ by:

$$\rho_g(R) \propto n^\beta(R) \quad (39)$$

where β is a fitting parameter to the projected X-ray surface brightness of the cluster (Jones & Forman 1984). Assuming that the gas makes up a fraction of the total mass that is roughly constant throughout the cluster, we have for the cluster density ρ :

$$\rho(R) \propto \rho_g(R) \propto n^\beta(R) \propto \left(\frac{R}{R_c}\right)^{-3\beta} \quad R \gg R_c. \quad (40)$$

At large radius, ρ behaves with radius as equation (35) with $p = 3\beta$. Expressed in terms of the galaxy number density, equation (38) now scales, for collisions occurring at $R > R_c$, as:

$$\Delta^{cluster} \propto n^\beta n_c^{-\beta/2} \quad (41)$$

where n_c is the galaxy number density at radius R_c . We are led to two different cases: 1) collisions at a fixed density n within a given cluster, and 2) collisions at a fixed density averaged over a set of clusters. The scaling with density of the collision factor (41) will be quite different in these two cases. It is of interest of considering case 2) for the purpose of comparing the predictions of our model to observations, which we will present later in this paper. For example, (41) can be averaged, at a fixed galaxy number density n , over a set of clusters with varying 'central' density n_c . Bahcall (1979) shows that the distribution of Abell clusters decreases as a function of luminosity L according to a Schechter law

$$\Phi(L) \propto L^\alpha \exp\left(-L/L_a\right) \quad (42)$$

where L_a is the cluster optical luminosity within the Abell radius R_a . Assuming that the cluster optical luminosity is proportional to the galaxy number density, the central density

distribution of Abell clusters is

$$\Phi(n_c) \propto n_c^\alpha \exp\left(-n_c/n_a\right) \quad (43)$$

with $n_a \propto L_a/R_a^3$. The collision factor (41) averaged at fixed n over the distribution (43) for $n_c \geq n$ is then

$$\langle \Delta^{cluster} \rangle \sim n^\beta \langle n_c^{-\beta/2} \rangle \quad (44)$$

where $\langle \rangle$ denotes average over (43):

$$\langle n_c^{-\beta/2} \rangle = n_a^{-\beta/2} f(n/n_a) \quad (45)$$

where the function $f(x)$ is defined by:

$$f(x) = \left[\frac{\int_x^\infty t^{\alpha-\beta/2} \exp(-t) dt}{\int_x^\infty t^\alpha \exp(-t) dt} \right]. \quad (46)$$

$f(n/n_a)$ is dominated by the contribution of densities $n \sim n_a$ and behaves roughly as $(n/n_a)^{-\beta/2}$. Thus, (44) becomes

$$\langle \Delta^{cluster} \rangle \sim n^{\beta/2}. \quad (47)$$

In the following, we will use equation (47) for the purpose of evaluating the morphology-density relation in our model.

3.5. Comparison between the field and clusters

Several differences in the collision factor in the two environments considered originate from the different scalings used in these two cases:

- In the field, the collision factor depends essentially on the present day density of the universe, whereas in clusters, Δ is a function of the local density ρ . As a consequence, the energy exchange in clusters are generally much more important than in the field, and depend on the environment, since they increase with increasing density.

- In the field, collisions at high redshift are more efficient (in terms of the fractional rate of change of binding energy per collision (28)) than recent collisions. This is a direct consequence of the strong increase of the average density of the universe with redshift. This effect overrules the fact that collisions for galaxies with small radii (that is, at a given mass, those formed at high z_{nl} - equation (25)), are less efficient than for large galaxies. Thus, at a given mass, (30) implies that the higher the redshift of formation of a galaxy, the higher the collision factor. On the contrary, in a given cluster, the fractional rate of change of binding energy per collision (32) is constant in time, as soon as the cluster has virialized. At a given mass, the higher the redshift of formation of the galaxy, the lower the galaxy radius (25), and the lower the collision factor (33).

4. A phenomenological definition of Hubble types

4.1. Angular momentum and morphological types

A key phenomenon to understand when elaborating a consistent picture of galaxy formation is the angular momentum history of galaxies of various Hubble types. It is believed that the angular momentum of primeval galaxies originates from tidal torques induced by the presence of neighboring galaxies (Doroshkevich 1970; White 1984). At the early epochs of galaxy formation, protogalaxies indeed interact strongly with their surroundings. The angular momentum acquired at that time may have a definitive impact on galaxy morphologies. It would be however misleading to associate isolated structures, i.e., objects that are less subject to tidal torquing, with elliptical galaxies. Following this argument, one would then expect to find fewer low-spin galaxies in high density environments, a prediction in disagreement with the observation of an increasing fraction of early-types when one goes towards richer clusters (Dressler 1980).

It is unlikely that the large amount of spin needed to rotationally support fully grown disk galaxies may be explained solely by the primordial angular momentum (e.g., Barnes & Efstathiou 1987). Dissipative collapse of the radiatively cooled gas in a dark halo must be invoked. Assuming that the gas angular momentum is conserved during the collapse phase, one needs a collapse by a factor 10 to achieve rotational support (Fall & Efstathiou 1980). A large initial angular momentum is not a sufficient condition for the formation of disk galaxies, as various processes, such as tidal interactions occurring during the lifetime of a galaxy, may induce angular momentum transport from the galaxy center to the outer parts. Thus an initially rapidly rotating protogalaxy could end up as an elliptical galaxy. Recent simulations indeed find an extremely high efficiency of angular momentum transfer resulting in disks that are far too small. Even if the gas starts to sink towards the galactic center, the settlement into a disk structure requires a gentle infall that could be perturbed by the tidal field. Indeed, modeling of disk formation as well as chemical evolution models (Lacey & Fall 1983; Rocca-Volmerange & Schaeffer 1990) supports slow disk formation via gas infall from a pre-existing halo. Spirals are found to have a relatively constant rate of star formation over the past 10 Gyr, and infall provides a possible means of regulating the gas supply and maintaining the disk in a state of marginal gravitational instability (Sellwood & Carlberg 1984). However, as soon as a protogalactic cloud collapses and decouples from the universal expansion, collisions are expected to occur due to its relative velocity with respect to other clouds. Virialization should be effective within one or a few Hubble times after formation and collisions during this period are not expected to greatly modify the final structure. More recent collisions, however, would inhibit the gentle infall of the gas needed to allow for the formation of a disk. Clearly, the collision history over the whole life of a galaxy since its epoch of formation should be taken into account if one wants to predict its morphological type.

4.2. Definition of Hubble types

The previous discussion leads us to define, in the framework of our model, the morphological types as follows: the condition for a spiral galaxy to form out of a cloud that first became non-linear at a redshift z_{nl} is to experience few, if any, collisions between the epoch of its formation and the epoch under consideration. Strong collisions will, on the contrary, prevent the gas from settling into a disk and allow for tidal exchanges that average out angular momentum. We shall consequently assume that a sizeable number of collisions between z_{nl} and the epoch characterized by z leads to the formation of elliptical galaxies. These rules for the formation of morphological types can be expressed in terms of conditions on the collision factor Δ , for field and/or cluster galaxies. At any redshift z , the collision factor for a spiral galaxy is assumed to satisfy the following condition:

$$\Delta(z) < \Delta_{th}^{spi} \quad (\text{Spiral}) \quad (48)$$

where Δ_{th}^{spi} is a redshift independent threshold on the collision factor and is a free parameter in this model. An elliptical galaxy will be such that:

$$\Delta(z) > \Delta_{th}^{ell} \quad (\text{Elliptical}) \quad (49)$$

where Δ_{th}^{ell} is another free parameter of our theory such that $\Delta_{th}^{ell} > \Delta_{th}^{spi}$. Finally, galaxies whose collision factors satisfy

$$\Delta_{th}^{spi} < \Delta(z) < \Delta_{th}^{ell} \quad (\text{S0}) \quad (50)$$

will define S0 galaxies in our model. Condensed objects, either having undergone few collisions, or which have collapsed too recently to form a disk, may be identified with damped $\text{Ly}\alpha$ clouds, $\text{Ly}\alpha$ forest or metal-line absorbers.

4.3. Conditions on the formation redshift of a galaxy

From the modeling of sections §3.2 and 3.3 (equations (30) and (33)), we now rewrite conditions (48)-(50) as conditions on the formation redshift of galaxies.

4.3.1. Field galaxies

In the field limit, conditions (48)-(50) become:

$$1 + z_{nl} < 1 + z_{spi}^f(z) \quad (\text{Spiral}), \quad (51)$$

$$1 + z_{nl} > 1 + z_{ell}^f(z) \quad (\text{Elliptical}) \quad (52)$$

and

$$1 + z_{spi}^f(z) < 1 + z_{nl} < 1 + z_{ell}^f(z) \quad (\text{S0}), \quad (53)$$

where z_{spi}^f and z_{ell}^f are limiting redshifts obtained by setting Δ equal to Δ_{th}^{spi} and Δ_{th}^{ell} , respectively, in equation (30). At $z = 0$ (the present epoch), and provided $z_{nl} \gg 0$, we find from (30) that

$$1 + z_{spi}^f(z = 0) \approx \left(\frac{\Delta_{th}^{spi}}{\Delta_*} \right)^{1/3} \quad (54)$$

and

$$1 + z_{ell}^f(z = 0) \approx \left(\frac{\Delta_{th}^{ell}}{\Delta_*} \right)^{1/3}. \quad (55)$$

In general, z_{spi}^f and z_{ell}^f are functions of the redshift z .

4.3.2. Cluster galaxies

Similar conditions on the redshift of formation can be obtained in the cluster limit from (33) and (48)-(50), provided that these conditions also hold in clusters. Those galaxies with

very large radii have stronger collisions and become ellipticals. Their redshift of formation, which at a given mass specifies their radius (25), is constrained by:

$$z_{nl} < z_{ell}^c(M, \rho) \quad (\text{Cluster elliptical}). \quad (56)$$

Spiral galaxies must be formed at redshift

$$z_{spi}^c(M, \rho) < z_{nl} \quad (\text{Cluster spiral}) \quad (57)$$

and S0 galaxies have a redshift of formation satisfying

$$z_{ell}^c(M, \rho) < z_{nl} < z_{spi}^c(M, \rho) \quad (\text{Cluster S0}). \quad (58)$$

It is to be noticed that, contrary to the conditions in the field, cluster ellipticals form at a somewhat lower redshift than spirals. This is due to the competition between two effects: galaxies that formed earlier will have experienced more collisions by today, but recently formed galaxies will have larger radii and the efficiency of collisions will be higher. In clusters, the latter effect predominates as discussed in section §3.5: at a given mass, recently formed cluster galaxies have a higher collision factor than older ones. The limiting redshifts z_{spi}^c and z_{ell}^c depend on the redshift z through (33). At the present time ($z = 0$), and provided $z_{nl} \gg 0$, we have from equation (33), with $\Delta^{cluster}$ equal to Δ_{th}^{spi} and Δ_{th}^{ell} respectively:

$$1 + z_{spi}^c(z = 0) \approx (1 + z_*) \left(\frac{M}{M_*} \right)^{-2/3} \left[\frac{\Delta_c}{\Delta_{th}^{spi}} \frac{\rho}{\rho_0} \left(\frac{v}{V_*} \right)^{-1} \right]^{-1} \quad (59)$$

$$1 + z_{ell}^c(z = 0) \approx (1 + z_*) \left(\frac{M}{M_*} \right)^{-2/3} \left[\frac{\Delta_c}{\Delta_{th}^{ell}} \frac{\rho}{\rho_0} \left(\frac{v}{V_*} \right)^{-1} \right]^{-1} \quad (60)$$

Note that Δ_{th}^{ell} and Δ_{th}^{spi} in (59) and (60) are not necessarily the same thresholds as in the field.

4.4. Summary

We have presented a model for galaxy collisions. Using a phenomenological definition of galaxy morphological types, we have obtained, for each type in the limit $z = 0$, a condition on the redshift of formation of a galaxy: equations (51)-(53) for field galaxies, (56)-(58) for cluster galaxies. These conditions depend on the thresholds Δ_{th}^{ell} and Δ_{th}^{spi} , which, as well as z_* , are free parameters of the model and scale all of the model predictions.

5. Determination of z_*

The conditions derived above on formation redshifts are all independent of the initial spectrum of fluctuations, which need not be specified ². Once an initial fluctuation spectrum is chosen, fluctuations at a given mass scale M are characterized by their effective height $\nu(M)$ in the matter density field linearly extrapolated until the present epoch:

$$\nu(M) \equiv \frac{\delta_c(1 + z_{nl})}{\sigma(M)} \quad (61)$$

where δ_c is the threshold on the linear density contrast required in structure formation theories, σ the linear variance of mass fluctuations on scale M - equation (15), and z_{nl} the redshift at which fluctuations on scale M virialize. The mass function (14) can then be expressed in terms of the height ν , given by (61), as:

$$\eta(M) = -\sqrt{\frac{2}{\pi}} \frac{\rho_0}{M^2} \frac{d \ln \sigma(M)}{d \ln M} \nu(M) \exp \left[-\frac{1}{2} \nu^2(M) \right]. \quad (62)$$

There is a consistency relation in order for the cooling constraint (23) that prevails at $M \approx M_*$ to provide the correct galaxy luminosity function $\Phi(L)$. The local luminosity

²We have introduced the CDM spectrum in §3.1 for the sole purpose of studying the variation of the mean galaxy mass \bar{M} with redshift.

function of the Stromlo-APM redshift survey (Loveday et al. 1992) is well fitted by a Schechter function:

$$\Phi(L) = \Phi_* \left(\frac{L}{L_*} \right)^{-\alpha} \exp(-L/L_*) \frac{dL}{L_*}, \quad (63)$$

where $\alpha \approx 0.97$ and $\Phi_* \approx 1.75 \times 10^{-3} \text{ Mpc}^{-3}$ (for $h = 0.5$). Assuming no luminosity evolution, we require that (63) and (62) match on the scale $M = M_*$, that is $\Phi(L)dL|_* = \eta(M)dM|_*$.

We get:

$$\nu_* \exp\left(-\frac{\nu_*^2}{2}\right) \approx -\frac{\Phi_*}{\exp(1)} \sqrt{\frac{\pi}{2}} \frac{M_*}{\rho_0} \frac{d \ln M}{d \ln \sigma(M)} \Big|_{M=M_*} \quad (64)$$

where $\nu_* \equiv \delta_c(1+z_*)/\sigma(M_*)$ and we have assumed a constant mass-to-light ratio. Solving equation (64), we find $\nu_* \approx 2.8$. Using CDM initial conditions (16) with a bias parameter $b = 1.67$ ($\sigma_8 \approx 0.6$) required to reproduce the abundance of the observed X-ray clusters at present in an $\Omega_0 = 1$ universe (White, Efstathiou & Frenk 1993), we have $\sigma(M_*) \approx 3.3$, from which we infer

$$(1+z_*) \approx 5.5. \quad (65)$$

Once we know z_* , the numerical values of the parameters of our model can be fixed. If we assume spherical symmetry, the radius of an $M_* = 10^{12} M_\odot$ galaxy formed at redshift z_* is $R_* = (3M_*/4\pi\rho_0 178(1+z_*)^3)^{1/3} \approx 50 \text{ kpc}$. Its velocity dispersion is $V_*^2 \approx 0.5GM_*/R_*$, implying $V_* \approx 200 \text{ km/s}$. We can then compute the values of Δ_* and Δ_c (equations (31) and (34)) provided we fix the quantities v_0 and $\varepsilon\rho_0/M_*$. The mean number density of M_* galaxies today, $\varepsilon\rho_0/M_*$, is inferred from the local luminosity function of the Stromlo-APM survey, given by Loveday et al. (1992): $\varepsilon\rho_0/M_* \approx \Phi_* \approx 1.75 \times 10^{-3} \text{ Mpc}^{-3}$. Taking $v_0 = 200 \text{ km/s}$, we find $\Delta_* = 1.75 \times 10^{-4}$ and $\Delta_c = 3.5 \times 10^{-4}$. The numerical values of the parameters of the model are summarized in table 3. The only remaining quantities to be determined in order to fully normalize our model are Δ_{th}^{spi} and Δ_{th}^{ell} .

6. Determination of Δ_{th}^{spi} and Δ_{th}^{ell}

The quantities Δ_{th}^{spi} and Δ_{th}^{ell} are fixed by requiring that the model produces the observed fractions of morphological types in the field today. These fractions can be evaluated in the following manner. Our phenomenological definition of morphological types implies definite conditions on the redshift of formation of galaxies. For example, field ellipticals are required to form at a redshift $z_{nl} \geq z_{ell}^f$ (equation (52)) in order to have experienced substantial energy exchange during collisions. The number density of objects of mass M which were already non-linear, with a mass between M/λ and M , at a redshift z' such that $z_{nl} > z' > z$ is given (Lacey & Cole 1993) by:

$$\eta(M, z, z') = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M^2} \frac{d \ln \sigma^{-1}(M)}{d \ln M} \frac{\delta_c(1+z)}{\sigma(M)} \exp \left[-\frac{1}{2} \left(\frac{\delta_c(1+z)}{\sigma(M)} \right)^2 \right] \times \text{erfc}(x) \quad (66)$$

where

$$x = \frac{\delta_c(z' - z)}{\sqrt{2} \sqrt{\sigma^2(M/\lambda) - \sigma^2(M)}} \quad (67)$$

and

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-u^2) du. \quad (68)$$

The erfc term represents the probability that an object present at z' still exists at redshift z . The present day fraction of ellipticals is thus given for $z' = z_{ell}^f$ by:

$$\mathcal{F}_{ell} = \frac{\int_{M_{inf}}^{M_{sup}} \eta(M, 0, z_{ell}^f) dM}{\int_{M_{inf}}^{M_{sup}} \eta(M, 0, 0) dM} \quad (\text{Field ellipticals today}). \quad (69)$$

The denominator of equation (69) is the number density of all condensed objects with mass between M_{inf} and M_{sup} , while the numerator counts only those objects that were formed before or at z_{ell}^f and that survived until the present epoch. Similarly, the fractions of spiral and S0 galaxies are given by

$$\mathcal{F}_{spi} = \frac{\int_{M_{inf}}^{M_{sup}} [\eta(M, 0, 0) - \eta(M, 0, z_{spi}^f)] dM}{\int_{M_{inf}}^{M_{sup}} \eta(M, 0, 0) dM} \quad (\text{Field spirals today}), \quad (70)$$

and

$$\mathcal{F}_{spi} = \frac{\int_{M_{inf}}^{M_{sup}} [\eta(M, 0, z_{spi}^f) - \eta(M, 0, z_{ell}^f)] dM}{\int_{M_{inf}}^{M_{sup}} \eta(M, 0, 0) dM} \quad (\text{Field S0s today}). \quad (71)$$

Equations (69)-(71) depend on Δ_{th}^{spi} and Δ_{th}^{ell} through the redshift cuts z_{spi}^f and z_{ell}^f . The observed fractions of morphological populations in the field today are $\approx 65\%$ spirals, $\approx 10\%$ ellipticals, and $\approx 25\%$ S0s (Dressler 1980; Postman & Geller 1984). For the purpose of evaluating (69)-(71), we use $\lambda = 2$ in equation (67) and $M_{inf} = 10^{10} M_{\odot}$. M_{sup} is given by the cooling constraint (23). The CDM spectrum (16) is used. We find in order to produce the correct abundance of morphological types that $\Delta_{th}^{spi} \approx 0.003$ and $\Delta_{th}^{ell} \approx 0.01$. The collision factor for field ellipticals is thus typically larger by a factor at least 3 than the one for ellipticals.

7. Redshift of formation of the morphological types in the field

Equations (51)-(53) give the typical redshift of formation of galaxies of different morphological types. Using the values derived previously for the thresholds on the collision factor, we find that typical field ellipticals form at $z_{nl} \gtrsim 2.5$ while typical field spirals form at somewhat smaller redshifts $z_{nl} \lesssim 1.5$. S0s form, in our scenario, at intermediate redshifts. Figure 2 illustrates this prediction. The number density of the three galaxy types in the field today ($z = 0$) are plotted as a function of the redshift of formation of the galaxies. The results have been obtained from equations (69)-(71) where the division by the total integrated galaxy number density has been omitted. The redshift cuts (51)-(53) have been used. The plot illustrates the main features of our phenomenological model: today's field ellipticals were formed at high redshift so as to experience efficient energy exchange through collisions. No galaxy forming at redshift lower than $z \simeq 2.5$ will end up as an elliptical by today. Conversely, recently formed galaxies have suffered little tidal disturbance and constitute today's spirals. Finally, galaxies that were born at intermediate redshifts define

the present day S0s.

8. Biasing between galaxy populations

Each of the conditions (51)-(53) implies a precise value of the height $\nu(M)$ of fluctuations condensing out of the primordial density field into galaxies. For example, we infer from (52) and (61) that field ellipticals of mass M will condense out of linear fluctuations of height ν such that

$$\nu > \nu_{ell}(M) \equiv \frac{1.68(1 + z_{ell}^f)}{\sigma(M)} \quad (72)$$

where we define the threshold ν_{ell} for ellipticals. For field spirals of mass M , we have

$$\nu < \nu_{spi}(M) \equiv \frac{1.68(1 + z_{spi}^f)}{\sigma(M)}. \quad (73)$$

where we define the threshold ν_{spi} for spirals. For M_* galaxies, we have $\nu_{ell} \approx 3$ and $\nu_{spi} \approx 2$. Equations (72) and (73) thus introduce a bias by effectively requiring that field ellipticals, spirals and S0s correspond to definite subsets of all mass condensations. By contrast with early theories of biased galaxy formation where ν was free to be chosen, the formation threshold in our model results from the modeling of the physical processes involved during galaxy formation.

9. The predicted morphology-density relation

As early as 1958, Abell noted that elliptical galaxies are more frequently found in the cores of dense clusters (Abell 1958; Morgan 1961) while spiral galaxies predominate in the outer parts of clusters and make up almost 70% of field galaxies. Dressler (1980) first noticed the existence of a relation between galaxy morphological types and the (projected) local density in which galaxies are found. Postman & Geller (1984) and Giovanelli,

Haynes & Chincarini (1986) have extended Dressler’s work to assess the existence of a morphology-3-D density relation in less dense environments such as for field galaxies and groups. Whether such a relation can be attributed to local properties of the environment, as claimed by Dressler, or can be related to initial conditions at birth (the ‘nature versus nurture’ controversy) has been an important issue assessed in more recent works (Whitmore & Gilmore 1991; Whitmore, Gilmore & Jones 1993). Several effects have been proposed that could induce evolution of the morphology of galaxies. Spitzer & Baade (1951) pioneered such an approach by suggesting that spirals could evolve into S0 galaxies by removal of their gas content. The dense cores of rich clusters provide an ideal environment: in these regions, ram-pressure stripping of the spiral interstellar medium by the intracluster gas is likely to occur (Gunn & Gott 1972). Direct mergers of disks (Mamon 1992) or tidal collisions (Spitzer & Baade 1951) are other likely effects. High resolution simulations suggest that compact sub- L_* S0s may form by the cumulative effect of tidal interactions on induced star formation and mass loss, so-called galaxy harassment (Moore et al. 1996). Competing theories have been developed by the advocates of the ‘nature’ hypothesis (Evrard et al. 1990). These various approaches suffer however from a number of problems: while the Evrard et al. (1990) model fails to reproduce the observed morphology-density relation for S0 and spiral galaxies, the result of removing the gas content of spiral galaxies by stripping would lead to S0-like galaxies, but not ellipticals, as stripping should not affect the stellar content of the stripped galaxy. Besides, Burstein (1979) observed that S0s seem to have thicker disks than spirals, which is difficult to reconcile with the hypothesis that spiral galaxies evolved into S0 by gas stripping. The present model lies somewhat in between the ‘nature’ and ‘nurture’ hypothesis. It is of interest to test its predictions for the morphology-density relation. We undertake this task in this section.

In order to evaluate the local morphology-density relation in our model, we need a continuous relation between the collision factor Δ and the density of the environment at

the present epoch. Scalings of Δ with density have been obtained in the two limiting cases of the field (equation (30), §3.2) and cluster (equation (47), §3.4) environments. However, galaxies which are in a cluster now have not necessarily always been in the same cluster environment. In an $\Omega_0 = 1$ CDM universe, clusters formed recently so that it seems fair to assume that energy exchange among colliding galaxies in a cluster supplement the early exchanges calculated for galaxies prior to cluster formation. In that case, the collision rate Δ for a given galaxy can be taken as

$$\Delta = \Delta^{field} + \gamma \Delta^{cluster} \quad (74)$$

where Δ^{field} is given by equation (30) and $\Delta^{cluster}$ by equation (33) using the scaling with the 3-D galaxy density n from equation (47). In (74), we have introduced the dimensionless parameter γ to take into account the fact that the thresholds Δ_{th}^{spi} and Δ_{th}^{ell} , that define the various morphological types through the conditions (48)-(50), may be different for cluster and field galaxies. According to (74), Δ^{field} dominates in the field, while the dominant contribution to the collision factor is $\Delta^{cluster}$ in high density environments. This defines a density n at which $\Delta^{cluster}$ starts to dominate over Δ^{field} . Observationally (Postman & Geller 1984), the morphological populations are rather constant up to $n \approx 1 \text{ Mpc}^{-3}$ ($h=0.5$). Accordingly, we fix the value of γ so that $\Delta^{cluster}$ starts to dominate over Δ^{field} at $n \approx 1 \text{ Mpc}^{-3}$. Numerically, using (30) and (47), we find $\gamma \approx 10^{-3}$. We have considered the case for an isothermal β -model for the cluster gas density with $\beta = 0.6$ (Jones & Forman 1984). The cluster velocity dispersion has been derived directly from the mass density profile using the virial theorem. Then, for $n \ll 1 \text{ Mpc}^{-3}$, the proportions of the various morphological types are the same as in the field, while for $n \gg 1 \text{ Mpc}^{-3}$, they are derived essentially from $\Delta^{cluster}$.

We can now evaluate the present day morphology-density relation, using equations (69)-(71) where the redshift cuts z_{ell} and z_{spi} are now derived from (74), with $\Delta = \Delta_{th}^{ell}$ and

Δ_{th}^{spi} respectively. Again, the CDM spectrum (16) is used and is normalized to the present abundance of X-ray clusters ($\sigma_8 \approx 0.6$). We use the mass function (66)-(67) with $\lambda = 2$. Figure 3 shows the morphology-density relation obtained from our modeling (solid lines) compared to the observed relation (histograms; Postman & Geller 1984) at the present epoch. A typical $1\text{-}\sigma$ error bar is shown for reference in the top panel (from Postman & Geller 1984). The model reproduces qualitatively the main features observed in the evolution with density of the proportions of morphological types: the spiral population decreases quite strongly in higher density environments, while this decrease is compensated by a corresponding increase of the elliptical and S0 populations. The agreement is actually rather good quantitatively also, at least for low and intermediate densities. At high densities ($n \approx 1000 \text{ Mpc}^{-3}$), the model overestimates slightly the fraction of spirals, while the predicted fraction of ellipticals is somewhat lower than observed.

Reproducing the observed morphology-density relation has required the adjustment of 4 free parameters of our model: the two thresholds Δ_{th}^{spi} and Δ_{th}^{ell} ; the parameter γ in (74); and the parameter β of the isothermal β -model in (47). The values of these parameters and the method used to determine them is summarized in table 4. The predictions of the model are quite sensitive to β and γ . For example, if we use the assumption that mass traces light in clusters, i.e. $\rho \sim n$, the model yields a decrease (an increase) of the spiral (elliptical) population at high density *sharper* than the ones observed. If we vary the value of γ , the density at which $\Delta^{cluster}$ starts to dominate over Δ^{field} is different from the observed value $n \approx 1 \text{ Mpc}^{-3}$. It is however remarkable that the model simultaneously reproduces the morphology-density for the three Hubble types considered. Once the model reproduces the observed morphology-density relation today, quantitative predictions can then be made for the evolution with redshift of the various fractions of morphological types, in the field as well as in clusters.

10. Evolution of galaxy populations with redshift

10.1. Redshift cuts

10.1.1. Field Galaxies

Conditions (48)-(50) on the collision factor Δ define the galaxy morphological types in our model. These conditions have been shown to be equivalent to conditions on the redshifts of formation of galaxies. The redshifts cuts on the redshift of formation, z_{ell} and z_{spi} , that delineate spirals from S0 and from ellipticals are function of the redshift, since the collision factor itself depends on the redshift. Our model has been normalized so as to reproduce the observed fractions of the different galaxy populations today. It is a natural development of the model to predict the evolution of these populations with redshift.

Equating Δ in equation (74) to the thresholds Δ_{th}^{ell} and Δ_{th}^{spi} , whose values have been determined previously, defines the two redshift cuts $z_{ell}(z)$ and $z_{spi}(z)$ at any redshift z . On figure 4, z_{ell} and z_{spi} are plotted (thin solid lines) as a function of redshift, for field galaxies. The field case is obtained by inserting the average galaxy density of the universe into equation (74) so that $\Delta^{cluster}$ is always small compared to Δ^{field} , whatever the redshift. The functions $z_{ell}(z)$ and $z_{spi}(z)$ define three regions in the (z, z_{nl}) space corresponding to specific ranges of values of Δ , as indicated on the figure. At a given redshift z , galaxies that went nonlinear at $z_{nl} > z_{ell}(z)$ are identified as ellipticals, those with $z_{nl} < z_{spi}(z)$ are spirals, and those whose redshift of formation lies in between the two redshift cuts are S0s. The thick solid arrow shows how a galaxy that formed at a given redshift z_{nl} (here $z_{nl} = 3.1$) would be identified at different redshifts: down to $z \simeq 2.7$, this galaxy is identified as a spiral, as long as energy exchanges induced by tidal collisions are not high enough to meet our morphology criterion for S0s. From $z \simeq 2.7$ to $z \simeq 1.8$, the galaxy is seen as an S0, before becoming an elliptical when the cumulative effect of collisions since formation is

important enough to have induced substantial energy exchange. In our scenario, galaxies thus evolve along the Hubble sequence as time goes on.

10.1.2. Cluster galaxies

For cluster galaxies, the redshift cuts $z_{ell}(z)$ and $z_{spi}(z)$ are given by solving equation (74) for $\Delta = \Delta_{th}^{ell}$ and Δ_{th}^{spi} respectively, this time with the average galaxy density of the universe replaced by the typical local galaxy density n of the cluster one wants to model. Figure 5 shows $z_{ell}(z)$ for $n = 500 \text{ Mpc}^{-3}$ compared to the field case. For a given n , there is a redshift at which Δ^{field} starts to dominate over $\Delta^{cluster}$. At high redshift, Δ^{field} dominates, as expected since clusters formed quite recently in a hierarchical universe. At low redshift, $\Delta^{cluster}$ dominates and $z_{ell}(z)$ is different from the field values. In figure 5, $\Delta^{cluster}$ dominates over Δ^{field} at $z \lesssim 1.5$. By construction, this redshift reflects the formation epoch of the cluster and is higher for higher values of n , since the cluster density reflects the density of the universe at the epoch of its formation.

Another interesting feature in figure 5 is that the redshift cut z_{ell} is lower in a high density environment than in the field, implying that cluster ellipticals form at a lower redshift than in the field, as tidal collisions in clusters are more efficient than in the field at converting disks into ellipticals.

10.2. Redshift evolution

At any redshift, the knowledge of the cuts $z_{ell}(z)$ and $z_{spi}(z)$ allows one to count the number density of the different galaxy populations. As usual, we evaluate the fractions of the various populations from equations (69)-(71) with $\eta(M, 0, z_{ell})$ replaced, this time, by $\eta(M, z, z_{ell})$, etc. All the results presented below assume the CDM spectrum (16)

normalized to $\sigma_8 = 0.6$.

10.2.1. *Definition of irregular galaxies*

So far, we have considered only three morphological types. However, observations at high redshift show the existence of a large population of so-called irregular galaxies (Brinchmann et al. 1997). These galaxies are morphologically perturbed, gas-rich, star forming galaxies with blue colors. There is a simple way to define ‘irregular’ galaxies in our model. Among all galaxies having experienced substantial energy exchanges during tidal collisions (essentially those that are ellipticals today), we can differentiate between those galaxies that have relaxed by the time characterized by redshift z and those that have not. The motivation for such a distinction is that unrelaxed galaxies may exhibit the disturbed morphology of observed irregulars, while relaxed galaxies are expected to show a smoother stellar distribution, as in ellipticals. Moreover, tidal collisions are likely to induce star formation in the interacting galaxies, and we can anticipate that the ‘irregulars’ of our model should be star-forming galaxies and have blue colors, in qualitative agreement with observations. However, part of the galactic gas is likely to be removed during collisions, and it is not clear whether ‘irregulars’ in our model will have retained enough gas to form stars. Clearly, modeling of the effect of collisions on star formation and the physics of the gas is required. Such models are beyond the scope of the present paper and we shall restrict ourselves to the present definition.

Our definition of ‘irregular’ galaxies thus amounts to introduction of a new redshift cut, z_{irr} . It is fixed by requiring that the elapsed time between the epoch of the last collision and the epoch characterized by z equals a typical time-scale for relaxation. Those galaxies that were formed at redshift higher than z_{irr} will have relaxed by redshift z and will be identified as ellipticals. In the opposite case, these galaxies will be ‘irregulars’. In practice,

we assume that relaxation occurs on a time-scale t_{relax} typically of the order of a dynamical time-scale t_{dyn} , that is

$$t_{relax} \sim t_{dyn} \propto \frac{1}{\sqrt{G\rho}} \propto (1 + z_{nl})^{-3/2} \quad (75)$$

where ρ is the density of the universe at the time of formation z_{nl} . The epoch of the last collision is more problematic to evaluate. Actually, in our picture, they may not be any *epoch of a last collision* as collisions continuously occur over the lifetime of a galaxy. However, the bulk of collisions must occur at high redshift when the density of the universe is high. The redshift z_{ell} (or a fraction of it) may be taken as the ‘redshift of last collision’, though more minor collisions may occur at lower redshift. There is some freedom in the choice of this epoch so that the definition of irregulars in our scenario is somewhat loose.

10.2.2. *Evolution of field galaxies*

Figure 6 shows the evolution of the population content for field galaxies. The main observed trends in the evolution of the galactic content of the universe are reproduced by our model: the fraction of field spirals decreases with increasing redshift as collisions in the past were more efficient at disrupting disk-forming galaxies. They make up 65% of the galaxy population today and only about 15% at redshift 3. The fraction of ellipticals rises smoothly from about 10% today to about 20% at redshift ~ 0.5 , then starts decreasing while the fraction of ‘irregulars’ takes over rapidly and starts to dominate at redshift $z \sim 1$. We note that the substantial rise of the ‘irregular’ fraction at redshift $z \sim 0.5$ is, at least qualitatively, consistent with a recent analysis of the peculiar/irregular populations of the CFRS and LDSS redshift surveys (Brinchmann et al. 1997).

10.2.3. Evolution of cluster galaxies

Figures 7 and 8 show the evolution of the galaxy population with increasing density of the environment. We take $n = 10 \text{ Mpc}^{-3}$ in figure 7 and $n = 500 \text{ Mpc}^{-3}$ in figure 8. At high redshift, where Δ^{field} dominates over $\Delta^{cluster}$ in equation (74), the fractions of galaxy types are essentially the same as in the field. Going towards lower redshifts, the fraction of spiral galaxies increases less rapidly than in the field, peaks around $z \sim 0.3$ (figure 7) and steadily drops from 55% to 50% at the present time. This decrease is compensated by a corresponding increase of the S0 and elliptical fractions, while the irregular population is quite unaffected. Thus, the cluster core is depleted from its spiral population as a direct consequence of the high density in the core, while it is populated by S0 and ellipticals. In figure 8, similar behavior is noticed: the predicted fractions start to depart from the field fractions at redshift $z \sim 1.5$, as expected from inspection of figure 5, with the peak in the spiral fraction occurring at a somewhat larger redshift than in figure 7. The drop of the fraction of spirals is more pronounced and goes from 45% at redshift $\sim 0.6 - 0.7$ to 25% at the present time. It is tempting to associate the decrease of the fraction of spirals from intermediate redshift to the present with the well known Butcher-Oemler effect (Butcher & Oemler 1978; Dressler et al. 1994). Indeed, tidal collisions occurring shortly after cluster formation might induce star formation in spirals, that then would appear similar to the blue Butcher-Oemler galaxies, while exhaustion and/or stripping of the gas content during subsequent collisions would transform spirals into S0s and ellipticals. Clearly, inclusion of collision induced star formation and gas physics in our model is needed for any detailed investigation of its predictions with respect to the Butcher-Oemler effect.

11. Conclusion

We have argued that galaxy collisions play a fundamental role in galaxy formation. We have built a model in which galaxy morphological types originate as a result of gravitational interactions with surrounding galaxies. Simple rules for energy exchange during collisions have been proposed that allow us to discriminate between different Hubble types: efficient collisions result in the disruption of disks and substantial star formation, leading to the formation of ellipticals; few or inefficient collisions in the past allowed a large gas reservoir to survive and form disks. Quantitative analysis of energy exchanges in the field and cluster environments have been presented. These analyses are based on the simulation results of Aguilar & White (1985).

Assuming that galaxy formation proceeds in a $\Omega_0 = 1$ Cold Dark Matter universe, our model both reproduces a number of observations and makes various predictions, among which are the redshifts of formation of the different Hubble types in the field. When the model is normalized to the present day abundance of X-ray clusters, we find that the amount of energy exchange needed to produce ellipticals implies that they formed by $z \gtrsim 2.5$ while spirals formed at $z \lesssim 1.5$.

The model also provides a natural basis for biasing between field spirals and ellipticals without requiring an *ad-hoc* identification of morphological types to peaks of different height in the initial density field. Field elliptical galaxies are found to be more biased with respect to mass than spiral galaxies by a factor ~ 1.5 . They preferentially form out of $\sim 3\sigma$ peaks, whereas spiral galaxies condense out of $\sim 2\sigma$ peaks.

Our formalism allows us to study galactic evolution in clusters. With the same collision rules as in the field, the model satisfactorily reproduces the morphology-density relation which spans a range of densities from 10^{-2} to 10^4 galaxies per Mpc^3 .

Finally, the predictive power of the model is exploited to predict the evolution of galaxy populations with redshift. The predicted trends are in good qualitative agreement with observations, both in the field and in clusters. Our modeling of collisions in clusters naturally gives rise to a Butcher-Oemler-like effect, observed in the predicted spiral population. However, inclusion in our model of gas physics and collision induced star formation is required for any detailed quantitative comparison of the predictions of our model to observations. Such a comparison will be undertaken in a forthcoming paper.

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Figures captions

Fig. 1.— Average galaxy mass \bar{M} (in M_\odot) as a function of redshift, with and without the cooling constraint (23). When cooling is included, \bar{M} is relatively constant up to $z \simeq 4$.

Fig. 2.— Number density of present day field galaxy populations as a function of the redshift of formation of galaxies. This graph shows the contribution of the three morphological types defined in our model to the present population of galaxies. Galaxies formed at redshift $z \geq z_{ell}^f$ become ellipticals by today. Galaxies formed at $z_{spi}^f \leq z \leq z_{ell}^f$ are present day S0s, while galaxies forming at $z \leq z_{spi}^f$ are today's spirals. The graph illustrates how morphological populations are defined and counted from a combined use of the mass function (66) and the predicted redshift cuts z_{ell}^f and z_{spi}^f .

Fig. 3.— Predicted local morphology-density relation (solid lines) compared to the data (histograms) of Postman & Geller (1984). A typical one standard deviation is shown on the top panel. The morphology-density relation is computed in our model by assuming that energy exchange in a cluster $\Delta^{cluster}$ supplement the early exchanges calculated for galaxies prior to cluster formation Δ^{field} . Scaling of $\Delta^{cluster}$ with the local galaxy density n is obtained by assuming that the cluster density distribution traces the gas density distribution. An isothermal β -model is assumed for the gas distribution.

Fig. 4.— Redshift cuts for the formation of Hubble types, z_{ell}^f and z_{spi}^f (thin solid lines), are plotted as a function of redshift in the field. z_{ell}^f and z_{spi}^f define three regions in the (z_{nl}, z) space, corresponding to three ranges of values of the collision factor Δ : ellipticals lie in the upper region, Spirals in the lower region, while S0s lie in the intermediate region. The solid

thick arrow shows how the morphological type of a galaxy that was born at redshift 3.1 evolves with redshift, under the effect of collisions.

Fig. 5.— The redshift cut z_{ell}^f in the cluster environment (dashed line) is compared to the same cut in the field (solid line). At high redshift, the collision factor (74) is given by the field limit Δ^{field} , while at lower redshift $\Delta^{cluster}$ starts to dominate over Δ^{field} and significant discrepancy with the field appears. Ellipticals in a cluster environment are able to form at lower redshift than in the field due to the higher efficiency of collisions in clusters.

Fig. 6.— Predicted fractions of galaxy populations in the field as a function of redshift. A new morphological type has been introduced and labeled as ‘irregulars’ (see text).

Fig. 7.— Predicted fractions of galaxy populations for $n = 10 \text{ Mpc}^{-3}$ as a function of redshift. At high redshift, predictions are the same as in the field, whereas, at low redshift, significant differences appear due to the different physics of collisions in denser environments.

Fig. 8.— Same as figure 7 for $n = 500 \text{ Mpc}^{-3}$. Increasing n accentuates the features of the evolution of the galaxy population at low redshift.

Table 1: Name and meaning of parameters used

Name	Meaning
R	Radius of the test galaxy
M	Mass of the test galaxy
V	Internal stellar velocity dispersion of the test galaxy
v	Relative velocity of encounters
ε	Mass fraction of the universe in galaxies
$\bar{\rho}$	Average density of the universe
f	Normalization of the collision efficiency of AG85 function f_E
$\dot{\Delta}$	Rate of change of binding energy due to collisions

Table 2: Rate of change of binding energy

Environment	Equation Used	Underlying Assumption
Cluster ($v \gg V$)	(10)	Impulse Approximation Straight line trajectories
Field ($v \sim V$)	(11)	Gravitational focusing (9)

Table 3: Numerical values of the model parameters

Parameter	Numerical Value	Units
Ω_0	1	...
H_0	50	km/s/Mpc
M_*	10^{12}	M_\odot
R_*	50	kpc
V_*	200	km/s
z_*	4.5	...
$\varepsilon\rho_0/M_*$	1.75×10^{-3}	Mpc^{-3}
v_0	200	km/s
f	30	...
Δ_*	1.75×10^{-4}	...
Δ_c	3.5×10^{-4}	...

Table 4: Tuned parameters for the morphology-density relation

Parameter	Numerical Value	Fixed By
β	0.6	Isothermal β -model
γ	10^{-3}	Galaxy populations at $n = 1 \text{ Mpc}^{-3}$
Δ_{th}^{spi}	0.003	Galaxy populations
Δ_{th}^{ell}	0.01	in the field today















